Geometric and Computational Aspects of Injection Molding*

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Abstract

In the manufacturing industry, finding an orientation for a mold that eliminates surface defects and insures a complete fill after the injection process is an important and difficult problem which has not previously been investigated formally. To date, only heuristic solutions have been proposed. We initiate the study of the geometrical and computational aspects of the injection molding process and present a linear time algorithm that solves this problem in two dimensions. We also characterize the object shapes (modeled as simple polygons) that can be 1-filled and relate fillability to known classes of polygons.

1 Introduction

'During injection, the mold is tilted into a favorable position that will eliminate surface defects such as bubbles and insure a complete fill. Mold orientation during fill is a cut-and-try process to find the most favorable position.' [10]

Our motive in studying injection molding is to understand the geometric and computational aspects that will allow us to remove the 'cut-and-try' from the process of finding a favorable mold position.

A mold, as defined in [4], refers to the whole assembly of parts that make up a cavity into which liquid is poured to give the shape of the required component when the liquid hardens. Given a three dimensional object as a mold, establishing whether there exists an orientation that allows the filling of the mold using only one pin gate (the pin gate is the point from which the the liquid is injected into the mold) as well as determining an orientation that allows the most complete fill are two major problems in the field of injection molding.

These problems seems difficult, and to date, only heuristics have been proposed as solutions these two problems [10], [12], [4], [7]. A thorough study of the two-dimensional case may shed some light on the difficulties involved in three dimensions. To this end, we solve the following problems of injection molding in the plane:

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1. Given an n-sided polygon with a point representing the pin gate and the direction of gravity, we determine in $O(n)$ time whether or not the polygon can be filled without forming air pockets. We extend this to polygons with holes.

2. Given a simple polygon, we determine in $O(n)$ time all the orientations that allow a filling of the polygon from one pin gate without causing any air pockets. For example, a convex polygon allows all orientations.

3. If we are only allowed to fill a polygon from one fixed direction, we find in $O(n \log n)$ time the direction that minimizes the number of air holes needed to fill the polygon.

4. We show that some well known classes of polygons can be filled with no air pockets using one pin gate, namely monotone polygons and weakly-edge visible polygons.

5. 'Molds for injection molding are most commonly of the clam-shell type to facilitate easy removal of the part.' [10] The corresponding two-dimensional object is a clam-shell polygon. A polygon is clam-shell if the 2 pieces of the mold can be removed by translation (these polygons are also referred to as 2-Moldable polygons [15]). We show that all clam-shell molds have an orientation that allow them to be filled with one pin gate, and we find all possible orientations in linear time.

6. We show that star-shaped polygons are not necessarily 1-fillable, but they are always 2-fillable. The direction of gravity as well as the placement of the pin gate that allows the 2-fillability of a star-shaped polygon are determined in $O(n)$ time.

7. We show that L-convex polygons are not necessarily 1-fillable, but they are always 2-fillable. The direction of gravity as well as the placement of the pin gate that allows the 2-fillability of an L-convex polygon are determined in $O(n \log n)$ time.

8. The class of polygons known as weakly-edge visible include the class of L-convex polygons (which are 2-fillable). We show however that weakly-externally visible polygons are not 2-fillable.

9. We prove that any polygon that is weakly internally visible from a chord is 2-fillable.

All proofs and details can be found in the full technical report [2].

2 Notation and Preliminaries

Let us first introduce some of the terminology we will be using in this paper.

We will denote the open interior of a simple polygon P by $int(P)$, the boundary by $bd(P)$, and the open exterior by $ext(P)$. The boundary is considered part of the polygon; that is, $P = int(P) \cup bd(P)$. A *chain* $C_{ij}(P) = (e_i, e_{i+1}, \ldots, e_j)$ is a sequence of edges on the boundary of P. Between any two points, s and t, on the boundary P, there is a clockwise polygonal chain from s to t, where the interior lies to the right of the chain and a counterclockwise chain where the interior lies to the left of the chain. Thus $C_{st}(P) \cup C_{ts}(P) = bd(P)$. We assume that the vertices of the polygon are in general position, i.e., no three vertices are collinear. This assumption simplifies our discussion but is not an inherent limitation of our methods.

The point on the polygon boundary from which the liquid is injected into the polygon is called the *pin gate*. We assume that the pin gate is the only point from which air is allowed to escape unless stated otherwise. A *venting hole* is a point from which only air and not any liquid is allowed to escape. We assume that neither the liquid being poured into the mold, nor the air in the mold are compressible. Finally, we assume that air cannot bubble out through the liquid.

By a *direction* we mean an equivalence class of *oriented* parallel lines. A *right (left) normal* to a given direction Θ is an equivalence class N of *oriented* parallel lines with the property that every member of N is orthogonal to Θ and oriented to the right (left) of Θ. The right normal of Θ will be denoted by $N^+(\Theta)$ and the left normal will be denoted by $N^-(\Theta)$.

By an *opposite or inverse direction* to a given direction Θ we mean an equivalence class I of *oriented* parallel lines with the property that every member of I is parallel to Θ but oriented in the opposite direction. We denote the opposite of a direction Θ by $opp(\Theta)$.

A given direction Θ will be specified by a point on a unit circle in the following way. Let C be a unit circle with center O. Let x be a point on the boundary of the circle such that ray(Ox) is parallel to and with the same orientation as the oriented line given by the direction Θ. Then direction Θ is represented by the point x.

Sometimes, we use the notion of direction to mean simply an equivalence class of parallel lines. In such cases we will explicitly use the term *unoriented direction*. An unoriented direction U will be specified by a pair of points p_1 and p_2 that are diametrically opposite on the unit circle, such that $L(\overline{p_1 p_2}) \in U$.

A *normal* to an unoriented direction U is an equivalence class N of *unoriented* parallel lines with the property that every member of N is orthogonal to U. Given an oriented direction Θ, we define the *unoriented normal* to Θ, denoted by $N(\Theta)$, as the unoriented direction that is orthogonal to Θ. Notice that the two points representing the unoriented normal divide the boundary of the unit circle into two semi-circles. We define the semi-circle containing the point representing the direction Θ as the *normal semi-circle* or the *normal half-plane* of Θ and denote it as $NH(\Theta)$. The semi-circle not containing the point representing the direction of Θ will be denoted by $NH_c(\Theta)$.

Given two points a and b on the unit circle, let $arc[a, b]$ denote the closed arc of the unit circle from a to b in clockwise direction and let $arc(a, b)$ denote the open arc from a to b in clockwise direction.

When liquid is injected into a polygon, the level of the liquid rises in the direction opposite that of gravity. The lowest horizontal line such that all the liquid in the polygon is contained below it is defined as the *level line*.

It is possible for the level line to be higher than the level of the liquid in some section of the polygonal mold. For example, the situation depicted in figure 1 can occur while the mold is being filled. Thus we define a *level chord* to be the horizontal chord representing the level of liquid in the subpolygon lying below the chord.

When the level line contains the pin gate, we say the polygon is filled. A region containing air in a filled polygon is called an *air pocket*. A polygon is said to be 1-fillable if there exists a pin gate and direction of gravity such that when the polygon is filled through the pin gate, there are no air pockets. If the polygon is not filled, the region inside the polygon, above the level line contains air. Similarly, the subpolygon containing the level chord, below the level line inside the polygon, contains air above the level chord. We call the highest point of an air pocket in a filled mold, the *peak* of the air pocket.

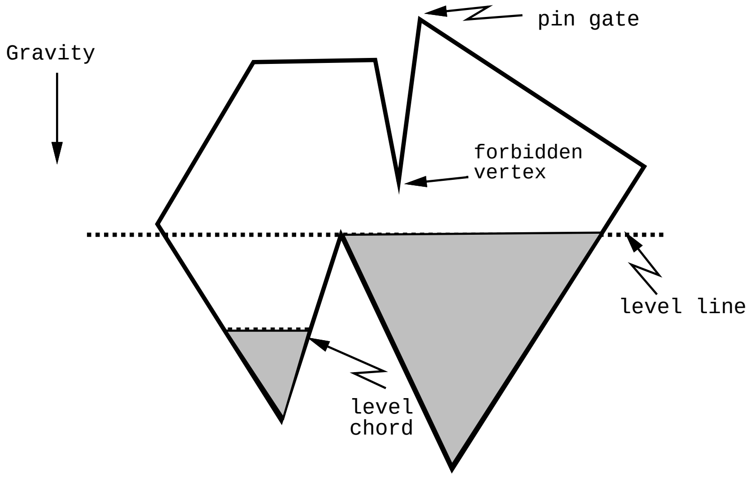

Figure 1:

3 Decision Problem

We begin by showing that given a simple polygon and a direction of gravity, in linear time, we can decide whether or not the polygon is 1-fillable with respect to the direction of gravity.

Let g be the point on the unit circle representing the direction of gravity. We observe the following.

Observation 3.1 *The peak of an air pocket is a local maximum with respect to the direction of gravity and is either a convex vertex v_i of the polygon P such that ray(v_iv_{i-1}) $\in NH(g)$ and ray(v_iv_{i+1}) $\in NH(g)$ or an edge e_i with endpoints v_i and v_{i+1} such that ray(v_iv_{i-1}) $\in NH(g)$ and ray($v_{i+1}v_{i+2}$) $\in NH(g)$.*

Observation 3.2 *The height with respect to gravity of the peak of an air pocket must be lower than the height of the pin gate.*

We now define what is known as a *forbidden vertex* and a *forbidden edge* of a polygon.

Definition 1 *A vertex v_i of the polygon P is forbidden if and only if it is a reflex vertex and ray(v_iv_{i-1}) $\in NH_c(g)$ and ray(v_iv_{i+1}) $\in NH_c(g)$.*

Definition 2 *An edge e_i with endpoints v_i and v_{i+1} of the polygon P is forbidden if and only if $L(e_i) \in N(g)$ and ray(v_iv_{i-1}) $\in NH_c(g)$ and ray($v_{i+1}v_{i+2}$) $\in NH_c(g)$.*

We can now state our first theorem:

Theorem 3.1 *A polygon is 1-fillable if and only if it contains no forbidden vertex or forbidden edge with respect to the direction of gravity.*

Corollary 3.1 *A polygon P is 1-fillable if and only if $\forall p \in P$, the geodesic path from p to the pin gate is monotonic with respect to the direction of gravity.*

Before we can prove theorem 3.1, we need a few lemmas.

Lemma 3.3 *A polygon with a forbidden vertex or forbidden edge will contain at least one air pocket when filled.*

Before showing the converse of the above lemma, we must establish a property of simple polygons. Given a point p [edge e] in the plane, let $h(p)[h(e)]$ denote the horizontal line containing p [e]. Let P be a polygon with two convex vertices c_1 and c_2 having the following properties. The y-coordinate of c_1 is greater than the y-coordinate of c_2; that is, c_1 is higher than c_2. The edges incident with c_1 lie below $h(c_1)$ and the edges incident with c_2 lie below $h(c_2)$.

Let x be the point of intersection between $h(c_2)$ and $bd(P)$ that is closest to c_2. Let C be the chain from c_2 to x such that all of the vertices lie below h. Such a chain must exist since x is the closest intersection point.

Lemma 3.4 *Chain C contains a vertex v or edge e such that the vertices adjacent to v lie above $h(v)$ or the vertices adjacent to e lie above $h(e)$.*

Lemma 3.5 *A filled polygon containing an air pocket must have a forbidden vertex or forbidden edge.*

The theorem 3.1 follows from lemma 3.3 and lemma 3.5.

A simple linear time algorithm for the decision problem arises from the theorem. By testing locally every reflex vertex, with respect to the direction of gravity, to determine whether or not it is a forbidden vertex, and testing every edge to determine whether or not it is a forbidden edge, we can determine if a polygon is 1-fillable with respect to the given direction of gravity. Using similar techniques, we also prove the following theorem.

Theorem 3.2 *A polygon with holes is 1-fillable if and only if it contains no forbidden vertex or forbidden edge with respect to the direction of gravity in either the polygon or any one of the holes.*

4 All Directions of Fillability

In the previous section, we showed that given a polygon and a direction of gravity, we can decide in linear time whether the polygon is 1-fillable with respect to gravity. The extension to polygons with holes is immediate, so for simplicity of exposition, we continue the discussion with simple polygons without holes.

Suppose that we are simply given a polygon and asked whether there exists a direction of gravity allowing the polygon to be 1-fillable with respect to that direction. In this section, we show that in $O(n)$ time the complete range of directions of gravity that allow the 1-fillability of a polygon can be determined. A direction that allows 1-fillability will be called a *feasible* direction of gravity.

Let us examine the set of directions of gravity that render a reflex vertex forbidden. Given a reflex vertex v_i of polygon P, the set of forbidden directions of gravity with respect to the given vertex, denoted by $F(v_i)$, is the following. $F(v_i) = \{\forall$ directions $g \| \text{ray}(v_i v_{i+1}) \in NH_c(g)$ and $\text{ray}(v_i v_{i-1}) \in NH_c(g)\}$

Let $g_l(v_i)$ be the direction such that $\text{L}(\overline{v_i v_{i-1}}) \in N(g_l(v_i))$ and $\text{ray}(v_i v_{i+1}) \in NH_c(g_l(v_i))$. Similarly, let $g_r(v_i)$ be the direction such that $\text{L}(\overline{v_i v_{i+1}}) \in N(g_r(v_i))$ and $\text{ray}(v_i v_{i+1}) \in NH_c(g_r(v_i))$. (Refer to figure 2).

Lemma 4.1 $F(v_i) = OA(g_r(v_i), g_l(v_i))$

Corollary 4.1 *The closed arc of the unit circle taken from $g_l(v_i)$ to $g_r(v_i)$ in a clockwise direction represents the set of feasible directions for gravity with respect to the vertex v_i.*

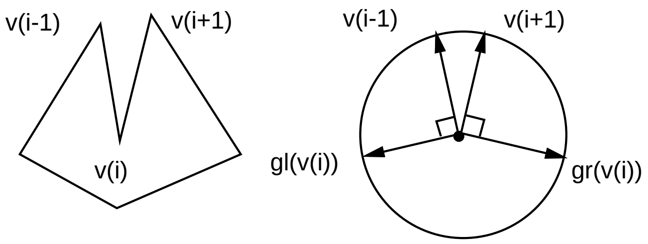

Figure 2:

We will denote the feasible directions of gravity with respect to the vertex v_i by $Feas(v_i)$. Let us now examine the set of directions of gravity that render an edge forbidden. Given an edge e_i of polygon P, the set of forbidden directions of gravity with respect to the given edge will be denoted by $F(e_i)$. From the definition of forbidden edge, we see that the only direction of gravity that renders an edge e_i with endpoints v_i and v_{i+1} forbidden is $N^+(v_iv_{i+1})$. Thus we have the following lemma.

Lemma 4.2 $F(e_i) = \{N^+(v_iv_{i+1}) \|$ *edge* e_i *is forbidden with respect to* $N^+(v_iv_{i+1})\}$

Corollary 4.2 *The feasible region of an edge* e_i*,* $Feas(e_i)$*, is represented by the boundary of the unit circle* $-F(e_i)$

The intersection of two arcs of a unit circle is either empty or an arc of the unit circle. Therefore, in linear time, we can compute and intersect all of the feasible regions with respect to the reflex vertices and the edges of a polygon P. Let $G(P)$ represent this intersection. If $G(P)$ is empty, then there is no direction of gravity that allows the 1-fillability of the polygon P and if $G(P)$ is not empty, then $G(P)$ represents all of the directions that allows the 1-fillability of P.

Theorem 4.1 *All the directions of gravity that allow 1-fillability of a polygon with holes can be found in* $O(n)$ *time.*

If a polygon is not 1-fillable from any direction, then filling from every direction will result in at least one air pocket. In such a situation, venting holes need to be placed in order to completely fill the polygon from one orientation. Minimizing the number of venting holes is desirable in this case.

For the next two lemmas, let P be a simple polygon that is not 1-fillable from any direction.

Lemma 4.3 *The number of local maxima in polygon* P *with respect to gravity = the number of forbidden vertices or edges - 1.*

Lemma 4.4 *The number of venting holes needed = the number of forbidden vertices and forbidden edges.*

Lemma 4.4 implies an algorithm to find the minimum number of venting holes needed to fill a polygon given that it must be filled in only one orientation, in $O(n \log n)$ time.

5 Fillability of Restricted Classes of Polygons

In the previous section, we showed that given a simple polygon, in linear time, we can find the complete range of orientations allowing 1-fillability. In this section we show the relationship between certain known classes of polygons and fillability.

Theorem 5.1 *A monotone polygon is 1-fillable if gravity is oriented in the direction of monotonicity.*

Let P be a weakly-edge visible polygon. Without loss of generality, let $\overline{v_1 v_2}$ be the edge from which the polygon is weakly visible. Let gravity g^* be oriented in the direction of the clockwise normal to ray$(v_1 v_2)$.

Theorem 5.2 *A weakly-edge visible polygon with gravity g^* is 1-fillable.*

Corollary 5.1 *Any polygon that is weakly visible from a chord is 2-fillable.*

The above corollary immediately implies the following.

Theorem 5.3 *A star-shaped polygon is not necessarily 1-fillable but always 2-fillable.*

A polygon is *clam-shell* if it can be partitioned into two chains $C_{ij}(P)$ and $C_{ji}(P)$ such that each chain can be removed by a single translation (not necessarily in a common direction). Clam-shell polygons were studied by [15] who proved the following.

Theorem 5.4 *[15] A polygon is clam-shell if and only if the boundary can be decomposed into two chains such that each is monotonic to an arbitrary direction. Clam-shells can be recognized in linear time.*

Thus we see that this class is a generalization of monotonic polygons. Before we can show that all clam-shell polygons are 1-fillable, we need to establish some key properties of clam-shell polygons. Let $C_{ij}(P)$ be monotonic with respect to some direction Θ. Let $D = \{\text{ray}(v_k v_{k+1}), i \leq k < j\}$. The monotonicity of $C_{ij}(P)$ implies that $\forall d \in D, d \in N(\Theta)$. But by definition 1 and definition 2, the feasible region of $C_{ij}(P)$ is the positive half-plane of the right normal of Θ. Thus, we have the following lemma.

Lemma 5.1 $Feas(C_{ij}(P)) \in NH(N^+(\Theta))$.

Theorem 5.5 *A clam-shell polygon is 1-fillable.*

A polygon P is *L-convex* if $\forall x, y \in$P$, \exists z \in P$ such that $\overline{xz} \in P$ and $\overline{yz} \in P$.

Lemma 5.2 *[6] If P is an L-convex polygon, it has the property that for every point $x \in P$, there exists a chord of the polygon containing x from which P is weakly visible.*

The lemma proved in [6] is more general and applies to L-convex sets. Corollary 5.1 immediately implies the following.

Theorem 5.6 *An L-convex polygon is not necessarily 1-fillable but always 2-fillable.*

Lemma 5.2 only proves the existence of the chord, but does not offer a method of computing such a chord given an L-convex polygon. We now present an $O(n \log n)$ algorithm to find such a chord. We first prove a few key lemmas.

Let x be a point inside polygon P. The *visibility polygon* from x, denoted by $VP(x, P)$, is the set of points in P visible from x. It is formally defined as

$$VP(x, P) = \{z \mid z \in P \text{ and } \overline{xz} \cap P = \overline{xz}\}$$

Let v_i be an arbitrary vertex of an L-convex polygon P. By lemma 5.2, there must be a chord containing v_i from which P is weakly visible. Let us denote this chord by $C(v_i)$. Thus, $C(v_i)$ must be contained in $VP(v_i, P)$. We will now show the relationship between a diagonal in $VP(v_i, P)$ and $C(v_i)$. Let b be a vertex of $VP(v_i, P)$. The chord $\overline{v_i b}$ is a chord in P and divides the polygon into two subpolygons, P_1 and P_2. If both P_1 and P_2 are weakly visible from $\overline{v_i b}$, then we have found $C(v_i)$. Otherwise, we will show that either P_1 or P_2 has to be weakly visible from $\overline{v_i b}$.

Lemma 5.3 *Either P_1 or P_2 or both are weakly visible from $\overline{v_i b}$.*

Notice that if the P_1 is not weakly visible from $\overline{v_i b}$ then $C(v_i)$ is contained in P_2 and vice versa. Thus, $C(v_i)$ is either a diagonal of the polygon P or contained between two consecutive diagonals. The question now becomes, how do we compute $C(v_i)$ if it is contained between two consecutive diagonals?

Consider the following situation. Let r and l be two consecutive vertices in $VP(v_i, P)$ such that P_1 is weakly visible from $\overline{v_i l}$ but not $\overline{v_i r}$ and P_2 is weakly visible from $\overline{v_i r}$ but not $\overline{v_i l}$.

Let $a_1, a_2, \ldots, a_m$ be the vertices in P_1 that are not weakly visible from $\overline{v_i r}$. They must be visible from $\overline{lr}$ since $C(v_i)$ is contained in the triangle(v_i, l, r). Similarly, let $b_1, b_2, \ldots, b_t$ be the vertices of P_2 that are not weakly visible from $\overline{v_i l}$. They must also be visible from $\overline{lr}$. Let a'_i be the point on $\overline{lr}$ farthest away from l, from which a_i is visible. Let $\overline{a} = \min_{1 \le i \le m} \|\overline{la'_i}\|$. Similarly, let b'_i be the point on $\overline{lr}$ farthest away from r, from which b_i is visible. Let $\overline{b} = \min_{1 \le i \le t} \|\overline{lb'_i}\|$. Let $S = \overline{a} \cap \overline{b}$.

Lemma 5.4 *S is not empty.*

Lemma 5.5 *For every $s \in S$, the polygon P is weakly visible from $\overline{v_i s}$.*

Lemma 5.3 and lemma 5.5 suggest the following algorithm.

Algorithm 2

1. Choose a vertex $v_1 \in P$.
2. Compute $VP(v_1, P)$ in $O(n)$ time using [5].
   [Let $\{v_1, \ldots, v_k\}$ denote the vertices of $VP(v, P)$]
3. Triangulate $VP(v, P)$ by inserting all diagonals $d_i = \overline{v_1 v_i}$, $2 \leq i \leq k$.
4. Let $l = 2$, $r = k$.
5. Let $s = \lfloor r/2 \rfloor$
6. Let P_1 be the polygon with vertices $\{v_1, v_l, v_{l+1}, \ldots, v_s\}$
7. Let P_1 be the polygon with vertices $\{v_1, v_s, v_{s+1}, \ldots, v_r\}$
8. If both P_1 and P_2 are weakly visible from d_s, exit with d_s. Use Avis and Toussaint [1] to verify weak visibility from d_s in $O(n)$ time].
9. If $r - l = 1$ goto step 12. [two consecutive diagonals at this point].
10. If P_1 is not visible from d_s, then $r = s$, goto step 5.
11. If P_1 is not visible from d_s, then $l = s$, goto step 5.
12. Compute $\overline{a} \cap \overline{b}$ in $O(n)$ time using Avis and Toussaint [1]. Pick any point s in $\overline{a} \cap \overline{b}$ and exit with $\overline{v_1 s}$.

The correctness of the algorithm follows from the discussion. Each step takes at most $O(n)$ time and we loop through steps 5 to 11 at most $O(\log n)$ times. Thus the total time complexity of the algorithm is $O(n \log n)$.

5.1 Weakly-Externally Visible Polygons

We have seen that the class of star-shaped polygons are 2-fillable. We have also seen that the class of L-convex polygons, which contains the class of star-shaped polygons is also 2-fillable. A natural question to ask is whether the class of weakly-externally visible polygons is 2-fillable, since the class of weakly-externally visible polygons contains the class of L-convex polygons (this is shown in [6]). A polygon is weakly-externally visible if for every point on its boundary there is an infinite ray emanating from that point in some direction that does not intersect the boundary. In fact, the class of weakly-externally visible polygons is in some sense the largest class of interest since it is the largest class of polygons that allows the pin gate to be placed anywhere on its boundary, if we image the pin gate as a line probe from infinity.

Consider the weakly-externally visible polygon in figure 3. No matter what direction we fill it from, at most only $1/4$ of the arms will be completely filled. But, $3/4$ of the arms cannot be filled from a second pin gate and orientation. Therefore, we have the following theorem.

Theorem 5.7 *A weakly-externally visible polygon is not necessarily 2-fillable.*

6 Conclusion

In this paper, we have presented a linear time algorithm to decide whether a polygon is 1-fillable in a given direction. We have also presented a linear time algorithm to find all the feasible directions

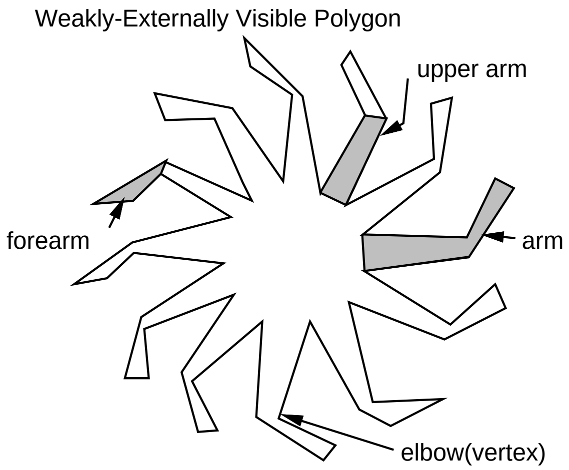

Figure 3:

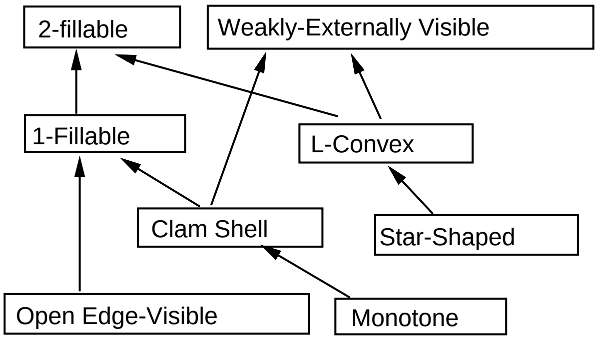

Figure 4:

from which a polygon is 1-fillable. Finally, we have related the notion of fillability to some know classes of polygons. We summerize those relationships by the chart in figure 4. These results have been extended to polyhedral molds (Bose, van Kreveld and Toussaint [3]). We conclude with some open problems.

1. Given a weakly externally visible polygon, is it k-fillable for some constant k? $(k > 2)$.
2. Given a simple polygon, can we find the minimum number of pin gates and orientations needed in order to fill the polygon?

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